Computer Algebra Systems Activity: Binomial Probability

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Topic: Binomial Probability

Ontario Expectations: (To be added when finalized by MOE.)

Notes to the Teacher:

a) This activity is designed to use the CAS on the TI-89 calculator to enhance understanding and instruction. Other CAS systems may be used in place of the TI-89. All screen shots are from the TI-89.

b) The activity is presented in a Teacher Version, with all screen shots and solutions present, as well as a Student Version, which can be duplicated and handed out to students.

Teacher Version:

Introduction: Two sports teams, the Polarbears and the Quahogs, are going to play off for the Big Trophy. Based on past performance, the probability of the Polarbears winning a given game is p, while that of the Quahogs winning is q, such that $p + q = 1$.

The teams may choose to play just one game, best of three, best of five, or best of seven.

a) As the number of games in the playoffs increases, what happens to the overall probability that the Polarbears will win the series? Does it increase, decrease, remain the same, or follow some other rule?

Your prediction: _____

b) Does your answer to part a) depend on the values of p and q? If so, how?

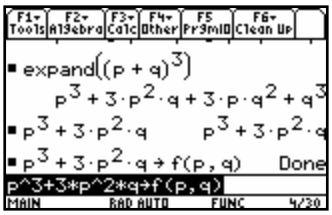
Your prediction: _____

1. Let $p = 0.6$. If only one game is played, what is the probability that the Polarbears will win?

[Answer: p, or 0.6.]

2. The probability distribution is binomial. The expansion of $(p + q)^n$, where n is the number of games played, models the probabilities that the Polarbears win or lose the series in various ways.

Enter $(p + q)^3$ into your TI-89. Select **expand(** from the **F2** menu, and expand the expression. The first term in the expansion gives the probability that the Polarbears will win all three games. The second term gives the probability that the Polarbears will win two out of three. The third term gives the probability that the Polarbears will win one out of three. The last term gives the probability that the Polarbears will win no games. Adding the first two terms gives the probability that the Polarbears will win the playoffs.



Delete the last two terms. Then, define $f(p, q)$ as the sum of the first two terms.

Evaluate $f(0.6, 0.4)$. What happened to the probability of the Polarbears winning the playoffs when the number of games was increased from 1 to 3?

[Answer: the probability increased from 0.6 to 0.648.]

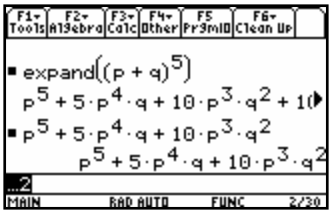
Does this result depend on the value of p? Evaluate $f(p, q)$ for values of p ranging from 0.1 to 0.9, in steps of 0.1. Record your results in the table shown.

p	**P(Polarbears Win)**
0.1	0.028
0.2	0.104
0.3	0.216
0.4	0.352
0.5	0.5
0.6	0.648
0.7	0.784
0.8	0.896
0.9	0.972

Does the value of p affect what happens as the number of games is increased from 1 to 3?

[Answer: If $p < 0.5$, the probability of winning the playoffs decreases. If $p = 0.5$, the probability of winning the playoffs stays the same. If $p > 0.5$, the probability of winning the playoffs increases.]

3. For a five-game series, the probabilities are given by the expansion of $(p + q)^5$. Repeat the analysis and the table for a five-game series.



p	P(Polarbears Win)
0.1	0.00856
0.2	0.05792
0.3	0.16308
0.4	0.31744
0.5	0.5
0.6	0.68256
0.7	0.83692
0.8	0.94208
0.9	0.99144

4. For a seven-game series, the probabilities are given by the expansion of $(p + q)^7$ Repeat the analysis and the table for a seven-game series.

p	P(Polarbears Win)
0.1	0.002728
0.2	0.033344
0.3	0.126036
0.4	0.289792
0.5	0.5
0.6	0.710208
0.7	0.873964
0.8	0.966656
0.9	0.997272

5. Compare your results for $p = 0.6$ for each of the cases above. What seems to be happening as the number of games increases?

[Answer: the probability of the Polarbears winning the playoffs is increasing as the number of games increases.]

6. Is it possible for the Polarbears to increase their probability of winning the playoffs to 1 by playing enough games? Explain.

[Answer: no. The value of the terms which are being discarded in each successive calculation will decrease as the number of games increases, but never to zero. Hence, the probability of the Polarbears winning the playoffs will always be less than 1.]

Student Version:

Introduction: Two sports teams, the Polarbears and the Quahogs, are going to play off for the Big Trophy. Based on past performance, the probability of the Polarbears winning a given game is p, while that of the Quahogs winning is q, such that $p + q = 1$.

The teams may choose to play just one game, best of three, best of five, or best of seven.

a) As the number of games in the playoffs increases, what happens to the overall probability that the Polarbears will win the series? Does it increase, decrease, remain the same, or follow some other rule?

Your prediction: _____

b) Does your answer to part a) depend on the values of p and q? If so, how?

Your prediction: _____

1. Let $p = 0.6$. If only one game is played, what is the probability that the Polarbears will win?

2. The probability distribution is binomial. The expansion of $(p + q)^n$, where n is the number of games played, models the probabilities that the Polarbears win or lose the series in various ways.

Enter $(p + q)^3$ into your TI-89. Select **expand(** from the **F2** menu, and expand the expression. The first term in the expansion gives the probability that the Polarbears will win all three games. The second term gives the probability that the Polarbears will win two out of three. The third term gives the probability that the Polarbears will win one out of three. The last term gives the probability that the Polarbears will win no games. Adding the first two terms gives the probability that the Polarbears will win the playoffs.

Delete the last two terms. Then, define $f(p, q)$ as the sum of the first two terms.

Evaluate $f(0.6, 0.4)$. What happened to the probability of the Polarbears winning the playoffs when the number of games was increased from 1 to 3?

Does this result depend on the value of p? Evaluate $f(p, q)$ for values of p ranging from 0.1 to 0.9, in steps of 0.1. Record your results in the table shown.

p	**P(Polarbears Win)**
0.1	
0.2	
0.3	
0.4	
0.5	
0.6	
0.7	
0.8	
0.9	

Does the value of p affect what happens as the number of games is increased from 1 to 3?

3. For a five-game series, the probabilities are given by the expansion of $(p + q)^5$. Repeat the analysis and the table for a five-game series.

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4. For a seven-game series, the probabilities are given by the expansion of $(p + q)^7$ Repeat the analysis and the table for a seven-game series.

p	**P(Polarbears Win)**
0.1	
0.2	
0.3	
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5. Compare your results for $p = 0.6$ for each of the cases above. What seems to be happening as the number of games increases?

6. Is it possible for the Polarbears to increase their probability of winning the playoffs to 1 by playing enough games? Explain.