

# Computer Algebra Systems Activity: Developing the Quadratic Formula

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**Topic:** Developing the Quadratic Formula

**Ontario Expectations:** (To be added when finalized by MOE.)

**Notes to the Teacher:**

a) This activity is designed to use the CAS on the TI-89 calculator to enhance understanding and instruction. Other CAS systems may be used in place of the TI-89. All screen shots are from the TI-89.

b) The activity is presented in a Teacher Version, with all screen shots and solutions present, as well as a Student Version, which can be duplicated and handed out to students.

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**Teacher Version:**

Note: This method should not be used as a substitute for developing the quadratic formula in the traditional way, but as an illustration of CAS.

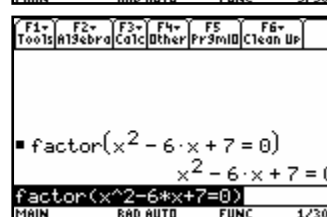
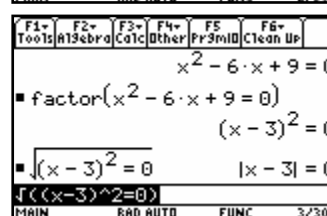
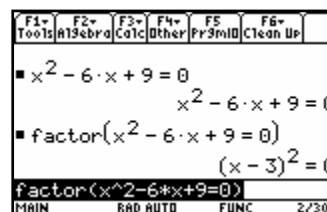
1. Solve the quadratic equation  $x^2 - 6x + 9 = 0$ .

Enter the equation into your TI-89. Factor the equation. Select **factor**( under the **F2** menu.

Copy the factored equation to the command line, or use the **ANS** key. Take the square root of both sides. Note that the only solution to the equation  $|x - 3| = 0$  is  $x = 3$ .

2. Now consider the equation  $x^2 - 6x + 7 = 0$ .

Inspection shows that this trinomial cannot be factored into two binomials using only integers. You can confirm this by attempting to factor the equation using your TI-89.



Try another approach. Subtract 7 from both sides. Think back to step (1) above. The left side can be turned into a perfect square by adding 9. You may have seen this pattern before: to make a perfect square, take half of the coefficient of the middle term, and square it. This is called "completing the square". Add 9 to both sides.

$$\begin{aligned} (x^2 - 6x + 7 = 0) - 7 \\ x^2 - 6x = -7 \\ (x^2 - 6x = -7) + 9 \\ x^2 - 6x + 9 = 2 \\ \text{ans}(1) + 9 \\ 2 \end{aligned}$$

Factor both sides, and then take the square root of both sides, following the pattern from step (1).

This time, there are two solutions to the absolute value equation:

$$\begin{aligned} \text{factor}(x^2 - 6x + 9 = 2) \\ (x - 3)^2 = 2 \\ \sqrt{(x - 3)^2} = 2 \quad |x - 3| = \sqrt{2} \\ \sqrt{\text{ans}(1)} \\ \sqrt{2} \end{aligned}$$

Either  $x - 3 = \sqrt{2}$  or  $x - 3 = -\sqrt{2}$ .

Therefore,  $x = 3 + \sqrt{2}$  or  $x = 3 - \sqrt{2}$ .

3. Consider the equation  $2x^2 + 12x + 9 = 0$ . You can easily confirm that this trinomial cannot be factored using integers. However, if you divide both sides by 2, you can follow a "complete the square" method, as in step (2). Divide both sides by 2.

$$\begin{aligned} 2x^2 + 12x + 9 = 0 \\ \frac{2x^2 + 12x + 9 = 0}{2} \\ \frac{2x^2 + 12x + 9}{2} = 0 \\ (2x^2 + 12x + 9 = 0) / 2 \\ x^2 + 6x + 9/2 = 0 \end{aligned}$$

Use **expand**( from the **F2** menu to simplify the division. Then, subtract  $9/2$  from both sides.

$$\begin{aligned} \text{expand}\left(\frac{2x^2 + 12x + 9}{2} = 0\right) \\ x^2 + 6x + 9/2 = 0 \\ (x^2 + 6x + 9/2 = 0) - 9/2 \\ x^2 + 6x = -9/2 \\ \text{ans}(1) - 9/2 \\ -9/2 \end{aligned}$$

Take half of the coefficient of the resulting  $x$  term, square it, and add to both sides.

$$\begin{aligned} (x^2 + 6x = -9/2) + 9/2 \\ x^2 + 6x + 9 = 9/2 \\ \sqrt{x^2 + 6x + 9 = 9/2} \\ |x + 3| = \frac{3\sqrt{2}}{2} \\ \sqrt{\text{ans}(1)} \\ \frac{3\sqrt{2}}{2} \end{aligned}$$

The absolute value equation has two solutions. You can solve for  $x$  in the usual way, or you can use **solve**( under the **F2** menu.

$$\begin{aligned} \text{solve}\left(|x + 3| = \frac{3\sqrt{2}}{2}, x\right) \\ x = \frac{3\sqrt{2}}{2} - 3 \text{ or } x = -\frac{3\sqrt{2}}{2} - 3 \\ \text{solve}(\text{ans}(1), x) \\ \frac{3\sqrt{2}}{2} - 3 \text{ or } -\frac{3\sqrt{2}}{2} - 3 \end{aligned}$$

4. You will now use the power of CAS to develop a solution for the general quadratic equation,  $ax^2 + bx + c = 0$ . Follow steps similar to those in part (3) to solve for  $x$ .

Divide both sides by  $a$ , and expand.

F1 Tools	F2 Algebra	F3 Calc	F4 Other	F5 Pr3mID	F6 Clean Up
$a \cdot x^2 + b \cdot x + c = 0$ $\frac{a \cdot x^2 + b \cdot x + c}{a} = \frac{0}{a}$ $\frac{a \cdot x^2 + b \cdot x + c}{a} = 0$					
ans(1)/a					
MAIN RAD AUTO FUNC 2/30					

F1 Tools	F2 Algebra	F3 Calc	F4 Other	F5 Pr3mID	F6 Clean Up
$a \cdot x^2 + b \cdot x + c = 0$ $\frac{a \cdot x^2 + b \cdot x + c}{a} = \frac{0}{a}$ $\frac{a \cdot x^2 + b \cdot x + c}{a} = 0$					
ans(1)/a					
MAIN RAD AUTO FUNC 2/30					

F1 Tools	F2 Algebra	F3 Calc	F4 Other	F5 Pr3mID	F6 Clean Up
$\frac{a \cdot x^2 + b \cdot x + c}{a} = 0$ $\frac{a \cdot x^2 + b \cdot x + c}{a} = 0$ $\frac{a \cdot x^2 + b \cdot x + c}{a} = 0$					
expand(ans(1))					
MAIN RAD AUTO FUNC 3/30					

Subtract  $c/a$  from both sides. Take half of the coefficient of the  $x$  term, square, and add to both sides. Then, factor both sides.

F1 Tools	F2 Algebra	F3 Calc	F4 Other	F5 Pr3mID	F6 Clean Up
$x^2 + \frac{b \cdot x}{a} + \frac{c}{a} = 0$ $\left(x^2 + \frac{b \cdot x}{a} + \frac{c}{a}\right) - \frac{c}{a} = 0 - \frac{c}{a}$ $x^2 + \frac{b \cdot x}{a} = -\frac{c}{a}$					
ans(1)-c/a					
MAIN RAD AUTO FUNC 4/30					

F1 Tools	F2 Algebra	F3 Calc	F4 Other	F5 Pr3mID	F6 Clean Up
$\left(x^2 + \frac{b \cdot x}{a} + \frac{c}{a}\right) - \frac{c}{a} = 0 - \frac{c}{a}$ $x^2 + \frac{b \cdot x}{a} + \frac{b^2}{4 \cdot a^2} = \frac{b^2}{4 \cdot a^2} - \frac{c}{a}$					
ans(1)+b^2/(2*a)^2					
MAIN RAD AUTO FUNC 5/30					

F1 Tools	F2 Algebra	F3 Calc	F4 Other	F5 Pr3mID	F6 Clean Up
$\left(x^2 + \frac{b \cdot x}{a} + \frac{b^2}{4 \cdot a^2}\right) = \frac{b^2}{4 \cdot a^2} - \frac{c}{a}$ $\frac{(2 \cdot a \cdot x + b)^2}{4 \cdot a^2} = \frac{b^2 - 4 \cdot a \cdot c}{4 \cdot a^2}$					
factor(ans(1))					
MAIN RAD AUTO FUNC 6/30					

Solve the absolute value equation for  $x$ .

F1 Tools	F2 Algebra	F3 Calc	F4 Other	F5 Pr3mID	F6 Clean Up
$\frac{(2 \cdot a \cdot x + b)^2}{4 \cdot a^2} = \frac{b^2 - 4 \cdot a \cdot c}{4 \cdot a^2}$ $\frac{(2 \cdot a \cdot x + b)^2}{4 \cdot a^2} = \frac{b^2 - 4 \cdot a \cdot c}{4 \cdot a^2}$ $\frac{(2 \cdot a \cdot x + b)^2}{4 \cdot a^2} = \frac{b^2 - 4 \cdot a \cdot c}{4 \cdot a^2}$					
f(ans(1))					
MAIN RAD AUTO FUNC 7/30					

F1 Tools	F2 Algebra	F3 Calc	F4 Other	F5 Pr3mID	F6 Clean Up
$\frac{(2 \cdot a \cdot x + b)^2}{4 \cdot a^2} = \frac{b^2 - 4 \cdot a \cdot c}{4 \cdot a^2}$ $\frac{(2 \cdot a \cdot x + b)^2}{4 \cdot a^2} = \frac{b^2 - 4 \cdot a \cdot c}{4 \cdot a^2}$ $\frac{(2 \cdot a \cdot x + b)^2}{4 \cdot a^2} = \frac{b^2 - 4 \cdot a \cdot c}{4 \cdot a^2}$					
solve(ans(1),x)					
MAIN RAD AUTO FUNC 8/30					

Note that you arrive at the familiar quadratic formula.

## Student Version:

Note: This method should not be used as a substitute for developing the quadratic formula in the traditional way, but as an illustration of CAS.

1. Solve the quadratic equation  $x^2 - 6x + 9 = 0$ .

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Factor both sides, and then take the square root of both sides, following the pattern from step (1).

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Use **expand**( from the **F2** menu to simplify the division. Then, subtract 9/2 from both sides.

Take half of the coefficient of the resulting x term, square it, and add to both sides. The absolute value equation has two solutions. You can solve for x in the usual way, or you can use **solve**( under the **F2** menu.

4. You will now use the power of CAS to develop a solution for the general quadratic equation,  $ax^2 + bx + c = 0$ . Follow steps similar to those in part (3) to solve for  $x$ .

Divide both sides by  $a$ , and expand.

Subtract  $c/a$  from both sides. Take half of the coefficient of the  $x$  term, square, and add to both sides. Then, factor both sides.

Solve the absolute value equation for  $x$ .

Note that you arrive at the familiar quadratic formula.