Computer Algebra Systems Activity: Developing the Quadratic Formula

© R. Meisel, Feb 19, 2005
rollym@vaxxine.com

Topic: Developing the Quadratic Formula

Ontario Expectations: (To be added when finalized by MOE.)

Notes to the Teacher:

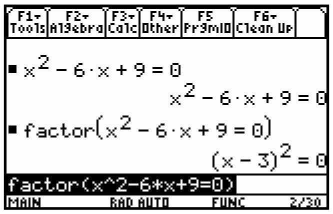
a) This activity is designed to use the CAS on the TI-89 calculator to enhance understanding and instruction. Other CAS systems may be used in place of the TI-89. All screen shots are from the TI-89.

b) The activity is presented in a Teacher Version, with all screen shots and solutions present, as well as a Student Version, which can be duplicated and handed out to students.

c) This material may be used freely by teachers in their classrooms. The copyright message must not be removed. Any other use or publication without the consent of the author is a breach of copyright.

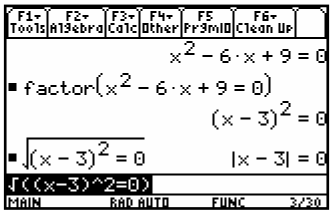
Teacher Version:

Note: This method should not be used as a substitute for developing the quadratic formula in the traditional way, but as an illustration of CAS.

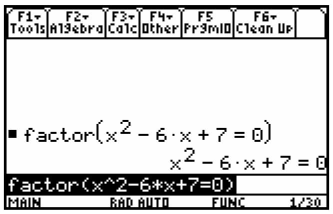


1. Solve the quadratic equation $x^2 - 6x + 9 = 0$.

Enter the equation into your TI-89. Factor the equation. Select **factor(** under the **F2** menu.

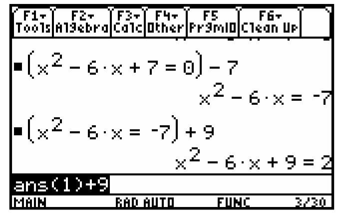


Copy the factored equation to the command line, or use the **ANS** key. Take the square root of both sides. Note that the only solution to the equation $|x - 3| = 0$ is $x = 3$.

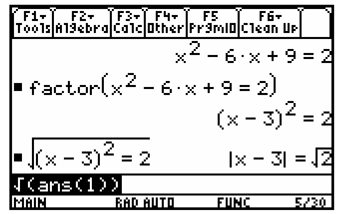


2. Now consider the equation $x^2 - 6x + 7 = 0$. Inspection shows that this trinomial cannot be factored into two binomials using only integers. You can confirm this by attempting to factor the equation using your TI-89.

Try another approach. Subtract 7 from both sides. Think back to step (1) above. The left side can be turned into a perfect square by adding 9. You may have seen this pattern before: to make a perfect square, take half of the coefficient of the middle term, and square it. This is called "completing the square". Add 9 to both sides.



Factor both sides, and then take the square root of both sides, following the pattern from step (1).

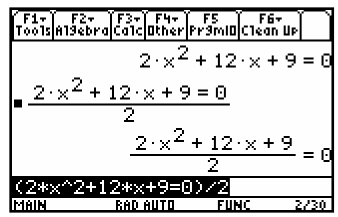


This time, there are two solutions to the absolute value equation:

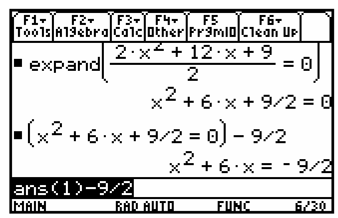
Either $x - 3 = \sqrt{2}$ or $x - 3 = -\sqrt{2}$.

Therefore, $x = 3 + \sqrt{2}$ or $x = 3 - \sqrt{2}$.

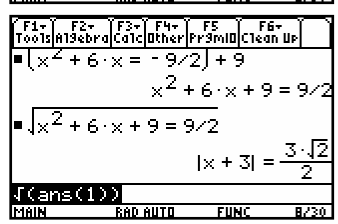
3. Consider the equation $2x^2 + 12x + 9 = 0$. You can easily confirm that this trinomial cannot be factored using integers. However, if you divide both sides by 2, you can follow a "complete the square" method, as in step (2). Divide both sides by 2.



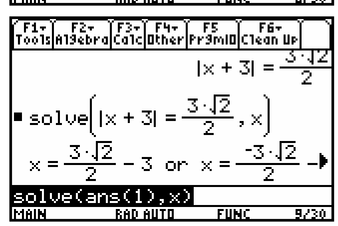
Use **expand(** from the **F2** menu to simplify the division. Then, subtract 9/2 from both sides.



Take half of the coefficient of the resulting *x* term, square it, and add to both sides.

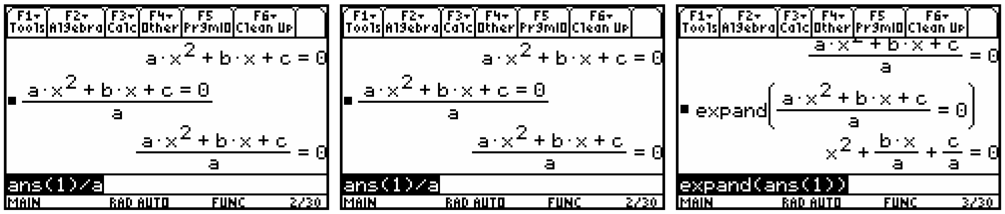


The absolute value equation has two solutions. You can solve for *x* in the usual way, or you can use **solve(** under the **F2** menu.

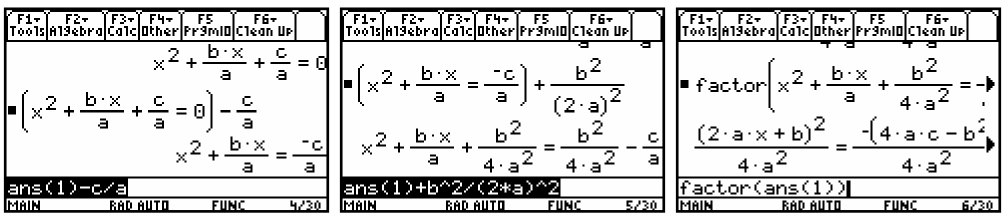


4. You will now use the power of CAS to develop a solution for the general quadratic equation, ax^2 + bx + c = 0. Follow steps similar to those in part (3) to solve for x.

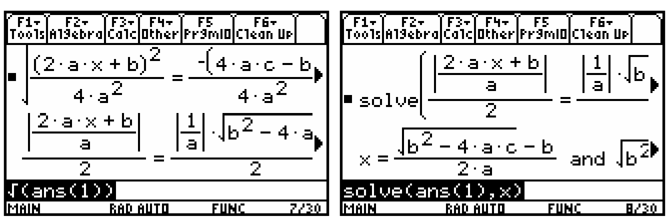
Divide both sides by a, and expand.



Subtract c/a from both sides. Take half of the coefficient of the x term, square, and add to both sides. Then, factor both sides.



Solve the absolute value equation for x.



Note that you arrive at the familiar quadratic formula.

Student Version:

Note: This method should not be used as a substitute for developing the quadratic formula in the traditional way, but as an illustration of CAS.

1. Solve the quadratic equation $x^2 - 6x + 9 = 0$.

Enter the equation into your TI-89. Factor the equation. Select **factor(** under the **F2** menu.

Copy the factored equation to the command line, or use the **ANS** key. Take the square root of both sides. Note that the only solution to the equation $|x - 3| = 0$ is $x = 3$.

2. Now consider the equation $x^2 - 6x + 7 = 0$. Inspection shows that this trinomial cannot be factored into two binomials using only integers. You can confirm this by attempting to factor the equation using your
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Factor both sides, and then take the square root of both sides, following the pattern from step (1).

This time, there are two solutions to the absolute value equation:

Either $x - 3 = \sqrt{2}$ or $x - 3 = -\sqrt{2}$.

Therefore, $x = 3 + \sqrt{2}$ or $x = 3 - \sqrt{2}$.

3. Consider the equation $2x^2 + 12x + 9 = 0$. You can easily confirm that this trinomial cannot be factored using integers. However, if you divide both sides by 2, you can follow a "complete the square" method, as in step (2). Divide both sides by 2.

Use **expand(** from the **F2** menu to simplify the division. Then, subtract 9/2 from both sides.

Take half of the coefficient of the resulting *x* term, square it, and add to both sides. The absolute value equation has two solutions. You can solve for *x* in the usual way, or you can use **solve(** under the **F2** menu.

4. You will now use the power of CAS to develop a solution for the general quadratic equation, ax^2 + bx + c = 0. Follow steps similar to those in part (3) to solve for *x*.

Divide both sides by a, and expand.

Subtract c/a from both sides. Take half of the coefficient of the *x* term, square, and add to both sides. Then, factor both sides.

Solve the absolute value equation for *x*.

Note that you arrive at the familiar quadratic formula.