Computer Algebra Systems Activity: Patterning

© R. Meisel, Feb 15, 2005
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Topic: Patterning and Algebra

Ontario Expectations: (To be added when finalized by MOE.)

Notes to the Teacher:

a) This activity is designed to use the CAS on the TI-89 calculator to enhance understanding and instruction. Other CAS systems may be used in place of the TI-89. All screen shots are from the TI-89.

b) The activity is presented in a Teacher Version, with all screen shots and solutions present, as well as a Student Version, which can be duplicated and handed out to students.

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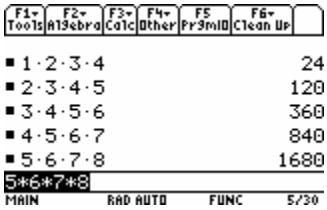
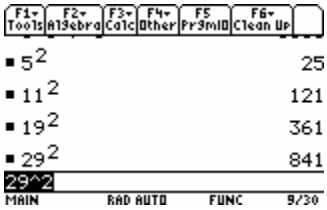
Teacher Version:

Introduction: Patterns that occur in series of numbers can be determined with pencil and paper or the use of a simple calculator. However, CAS allows you to verify that your pattern will work for an infinite series. In this exercise, you will use your calculator to help you find a pattern, and then use CAS to show whether the pattern works regardless of the beginning number chosen.

1. Find the product of the integers from 1 to 4. Then, find the product of the integers 2 to 5. Continue, starting one integer higher each time, until you have five products. Display your results in a table.



2. Look for a pattern in your results. Ensure that each of your results matches the pattern. Then, continue for three more steps, and check the pattern.
(Hint: If you can't find a pattern, decode this message by replacing each letter with the one before it in the alphabet: "dpotjefs trvbsft pg joufhfst"



[Answer: each number is one less than the square of an integer. The base follows the pattern $5 + 2(x - 1)$, where x is the term number.]

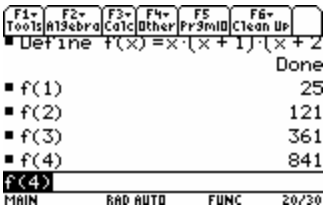
3. For the first term, the product is 1×2×3×4. Write an expression for the product of the x^{th} term.

[Answer: $x(x + 1)(x + 2)(x + 3)$]

4. Add 1 to this expression.

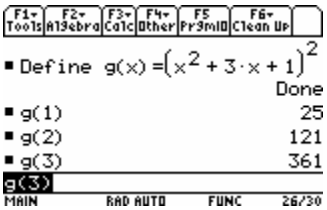
[Answer: $x(x + 1)(x + 2)(x + 3) + 1$]

5. Use the **define(** function under **F4** to define f(x) as the expression in part (4). Verify that f(x) generates the expected numbers for integers from $x = 1$ to $x = 5$.



6. Now you will use the CAS engine to show that the answer will be a perfect square regardless of the integer chosen for x. Select the **expand(** function under **F2** to expand f(x), and then use the **factor(** function under **F2** to factor the resulting expression.

7. Define g(x) as the factored expression from part (6). Verify that g(x) generates the expected numbers for integers from $x = 1$ to $x = 5$.



8. Extensions:

a) Does the pattern work for four consecutive negative integers?

b) How about a mixture of positive and negative integers? Before trying examples on your calculator, do a little thinking, and predict the results. Then, use your calculator to verify the results.

Student Version:

Introduction: Patterns that occur in series of numbers can be determined with pencil and paper or the use of a simple calculator. However, CAS allows you to verify that your pattern will work for an infinite series. In this exercise, you will use your calculator to help you find a pattern, and then use CAS to show whether the pattern works regardless of the beginning number chosen.

1. Find the product of the integers from 1 to 4. Then, find the product of the integers 2 to 5. Continue, starting one integer higher each time, until you have five products. Display your results in a table.

Integers	Product
1×2×3×4	

2. Look for a pattern in your results. Ensure that each of your results matches the pattern. Then, continue for three more steps, and check the pattern.

(Hint: If you can't find a pattern, decode this message by replacing each letter with the one before it in the alphabet: "dpotjefs trvbsft pg joufhfst"

Result	Pattern
1×2×3×4	

3. For the first term, the product is 1×2×3×4. Write an expression for the product of the x^{th} term.

4. Add 1 to this expression.

5. Use the **define(** function under **F4** to define f(x) as the expression in part (4). Verify that f(x) generates the expected numbers for integers from x = 1 to x = 5.

6. Now you will use the CAS engine to show that the answer will be a perfect square regardless of the integer chosen for x. Select the **expand(** function under **F2** to expand f(x), and then use the **factor(** function under **F2** to factor the resulting expression.

7. Define g(x) as the factored expression from part (6). Verify that g(x) generates the expected numbers for integers from $x = 1$ to $x = 5$.

8. Extensions:

a) Does the pattern work for four consecutive negative integers?

b) How about a mixture of positive and negative integers? Before trying examples on your calculator, do a little thinking, and predict the results. Then, use your calculator to verify the results.