Computer Algebra Systems: An Introduction

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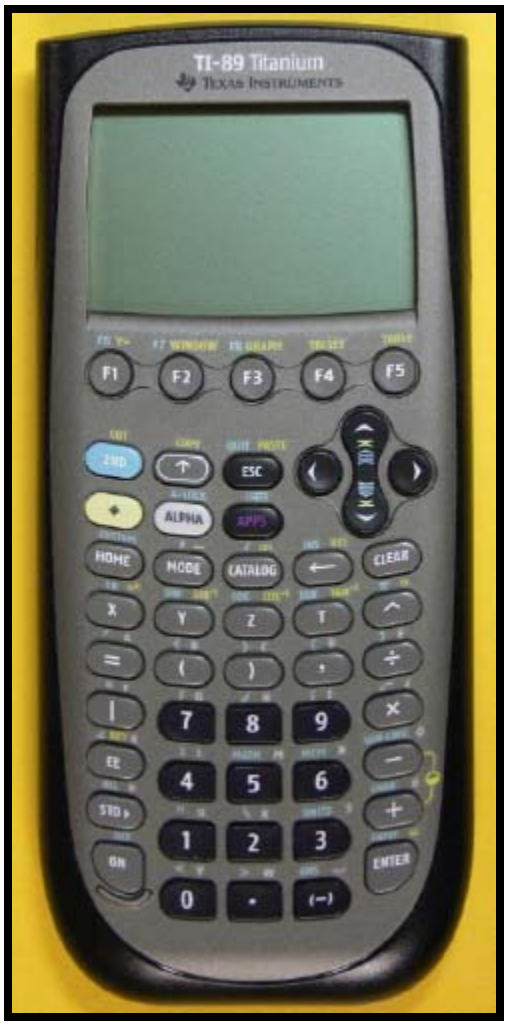
Topic: An Introduction to Computer Algebra Systems (CAS)

Notes to the Teacher:

This activity is designed to use the CAS on the TI-89/89T/92 calculator family to enhance understanding and instruction. All screen shots are from the TI-89.



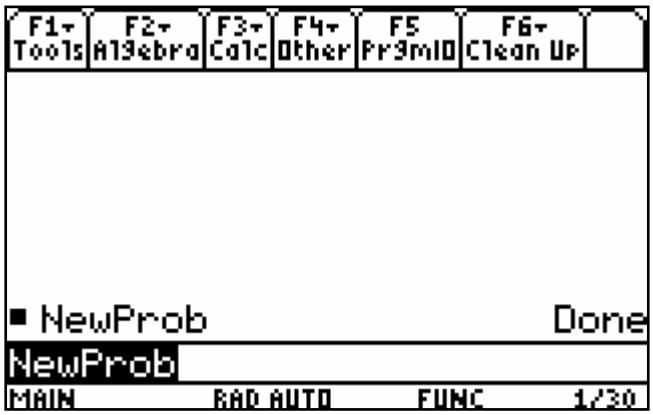
Computer Algebra Systems: An Introduction

What is a CAS?

A computer algebra system (CAS) is a program on a calculator or a computer that can perform algebraic manipulations. The CAS may also be known as a symbolic manipulator.

Starting a CAS

1. Press the **ON** key to turn the calculator on. If you don't see the Home screen, press the **HOME** key. The Home screen will appear.



2. Before using the CAS, it is wise to clear the calculator of any values left in the memory by a previous use or user. Press **2nd**, then **F1**, to access the **F6** or **Clean Up** menu. Select **2.NewProb**. Press **ENTER**.

3. Try some calculations to familiarize yourself with the keyboard and display.

a) $5(7 - 3)^3$ Answer: 320

b) $\pi(4.2)^2$ Answer: 55.42

c) $-\sqrt{25} - 3$ Answer: –8

Tech Tip: If you press a key in error, you can usually cancel its effect by pressing the **ESC** (Escape) key.

Keep in mind the difference between the negative key **(-)**, which makes a number negative, and the subtract key, which is used between two operands.

4. Investigate the **Function** keys at the top of the keypad.

a) Press **F1** to see the **Tools** menu.

b) Press **F2** to see the algebraic manipulations that the CAS can perform.

c) You can also use the left and right cursor keys to see the different function menus. Look at the selections under **F3**, the calculate menu, and **F4**. You will use some of these selections as you work through CAS tutorials.

d) If the cursor is in the command line, pressing the **CLEAR** key will delete the contents of the command line. If you move the cursor into the work space, pressing the **CLEAR** key will clear the line that the cursor is on.

The CAS will "remember" lines, even if they have disappeared off the top of the workspace. You can use the cursor keys to scroll to these hidden lines.

Simplifying or expanding expressions

1. Consider an algebraic expression such as $7x + 5y – 4x – 3y$.

Type the expression, and press **ENTER**.

Note that the CAS has collected the like terms for you.
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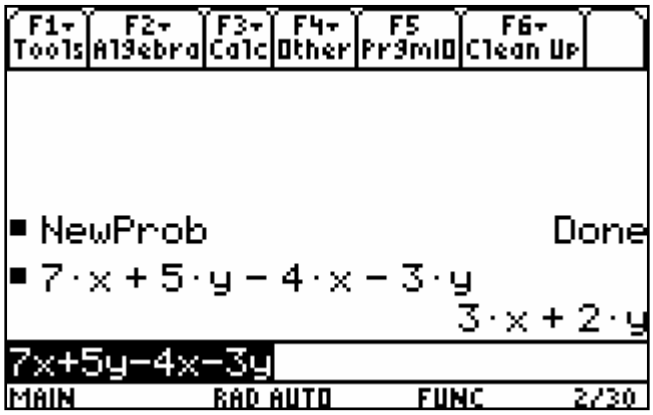


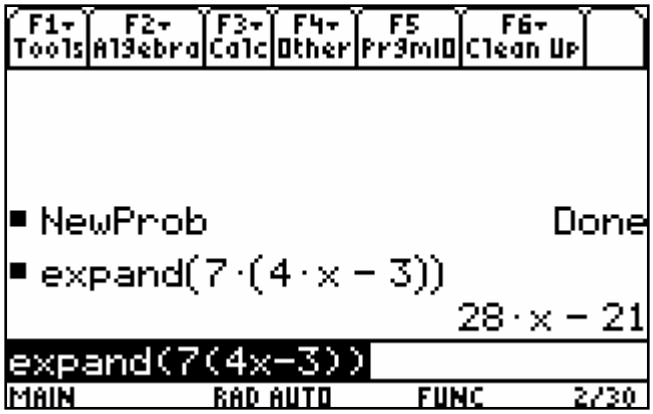
2. Try some other algebraic expressions. Include terms with exponents.

3. The CAS can expand algebraic expressions such as $7(4x – 3)$using the distributive property.

Press **F2**, and select **3:expand(**.

Type the expression, followed by a close bracket **)**.

• Press **ENTER**.



Note that nested brackets are required. If you get an error message while using this operation, check that you have the same number of open and close brackets, and that they are placed properly.

4. Try some other expansions. Include a product of binomials, such as $(2x – 3)(3x + 1)$, or even the product of a binomial and a trinomial such as $(5z – 2)(3z^2 + z – 4)$.

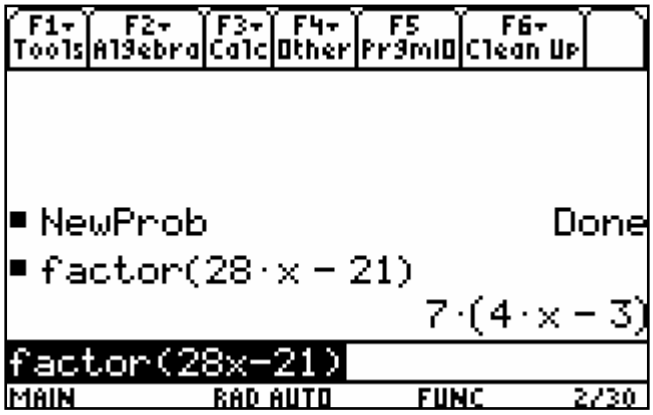
Factoring expressions

1. Press **F2**, and select **2:factor(**.

Type $28x + 21$, followed by a close bracket **)**.

Press **ENTER**.

Compare the result to the expansion in part 3 of the previous section.



2. Try factoring some other expressions, including $6x^2 – 7x – 3$.

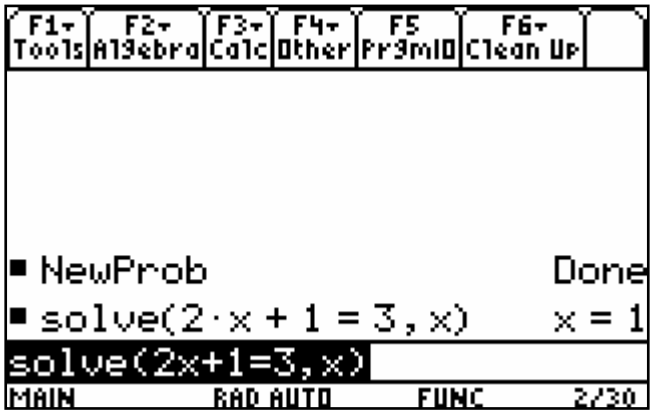
Solving equations

The CAS can solve equations.

1. Press **F2**, and select **1:solve(**.

Type $2x + 1 = 3$, then a comma, then an x followed by a close bracket **)**.

Press **ENTER**.



Note that the solution is displayed. Note, also, that you must specify the variable that you would like the CAS to solve for.

Try solving a few more equations.

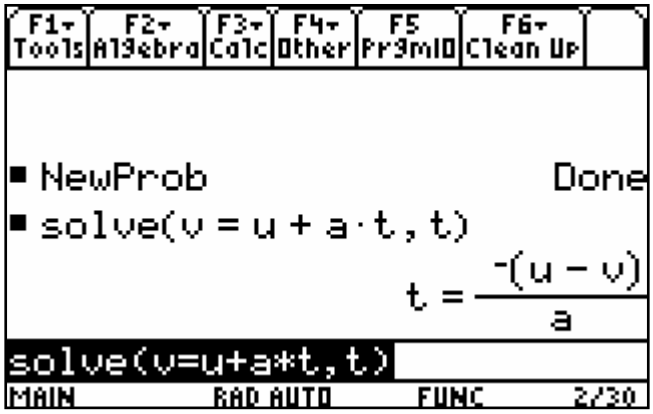
The CAS can solve for a variable in a formula. Consider the motion equation $v = u + at$. Suppose that you want to solve this formula for time, t.

If you inspect the calculator keyboard, you will note that x, y, z and t have dedicated keys. Other variables are entered by pressing the **ALPHA** key first. For example, to access a, press **ALPHA** , then =**.**

2. Press **F2**, and select **1:solve(**.

Type $v = u + a \times t$, then a comma, then t followed by a close bracket **)**.

Tech Tip: be sure to type the multiplication operation between the a and the t. Otherwise, the CAS will treat at as a single variable with the name at.



Press **ENTER**.

Tech Tip: the new form of the formula is sometimes displayed in an unusual manner.

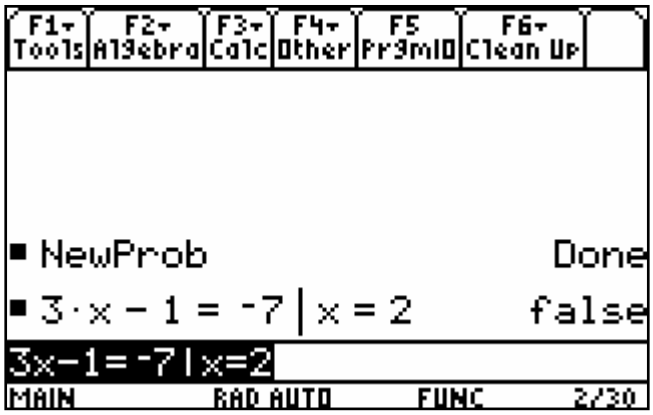
Try solving other formulas, such as $V = \pi r^2 h$ for r.

Checking the solution to an equation

You can use the CAS to test a solution to an equation. As an example, test whether $x = 2$ is a solution for the equation $3x - 1 = -7$.

1. Type $3x - 1 = -7$. Then, press the **|** (such that) key, followed by $x = 2$.

Press **ENTER**.



Note that the CAS returns a value of **false**. Hence, $x = 2$ is not a solution for the equation. Try the check again with $x = -2$. Note that this time the CAS returns a value of **true**. Hence, $x = -2$ is a solution.

Try checking a few more solutions for other equations.

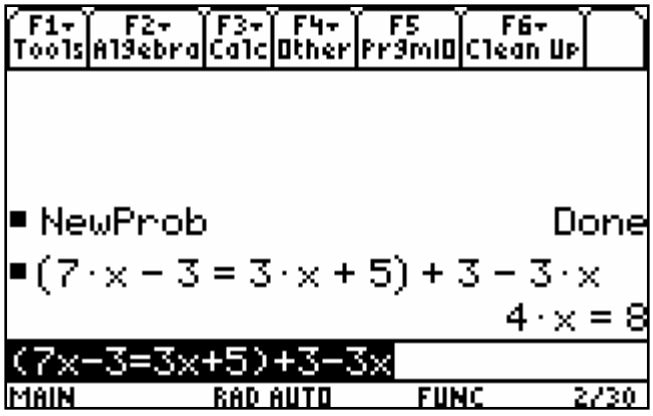
Operations on equations

You can use the CAS to apply the same operation to both sides of an equation. Consider the equation $7x - 3 = 3x + 5$. If you are solving this equation using pencil and paper, you begin by adding 3 to both sides and subtracting $3x$ from both sides. You can do this with the CAS.

1. Type the equation into the CAS, inside a set of brackets.

Outside the brackets, type + 3 – 3*x*. Press **ENTER**.

Tech Note: the syntax is somewhat different from that normally used in a pencil and paper approach.



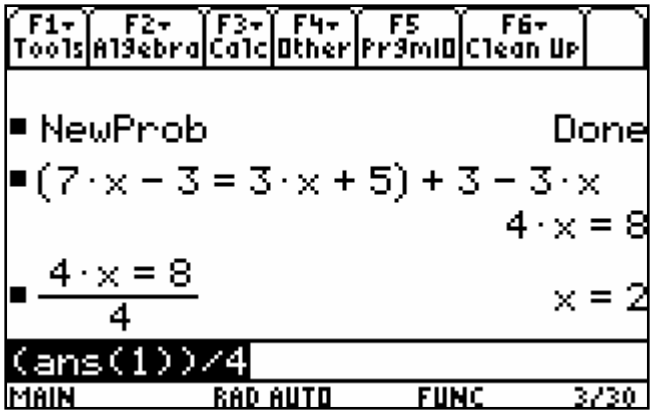
The next step is to divide both sides by 4. You can make use of the **ANS** key to do this.

2. Open a bracket.

Type **2nd**, then **(-)** to access **ANS**. This tells the CAS to use the answer from the previous operation.

Close the brackets.

Type ÷ 4.



Press **ENTER**. Note that the expected answer is displayed.

Copying and Pasting

If you have entered a complicated expression at some point, and need to use it again, you can avoid retyping by using the **COPY** and **PASTE** operations.

Use the cursor keys to scroll up to the expression that you want. Press the ♦ key, and then the ↑ key to access **COPY**.

Scroll back down to the command line. Press ♦ and **ESC** to access **PASTE**.

The expression is pasted into the command line.

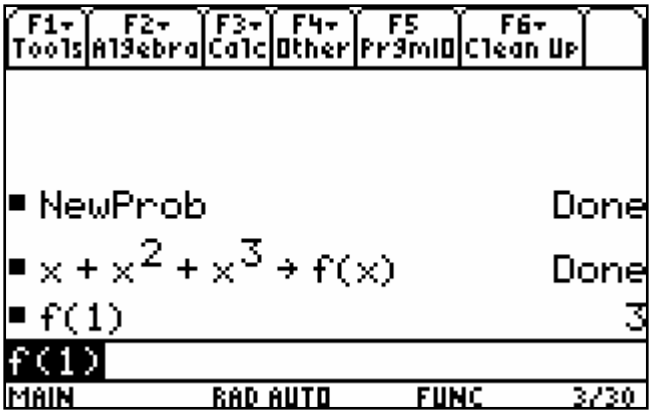
Defining functions

You can assign a definition to a function using the CAS.

As an example, define

$f(x) = x + x^2 + x^3$.

Type the expression on the right of the equals sign. Then, press the **STO** (or store) key. Type f(x). Remember that you need the **ALPHA** key to access f.



Press **ENTER**.

Type f(1), and press **ENTER**.

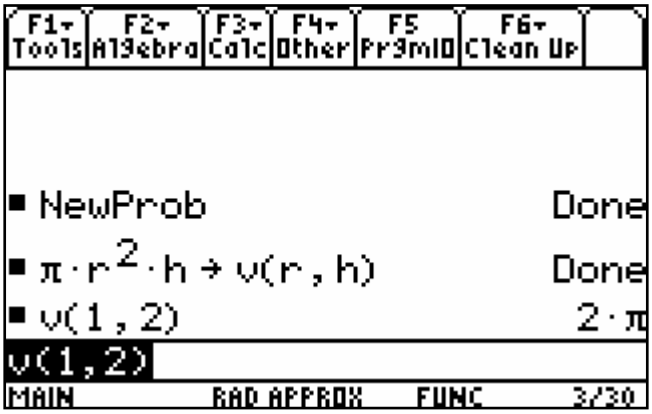
Evaluate the function for other values of *x*.

You can also define functions of more than one variable.

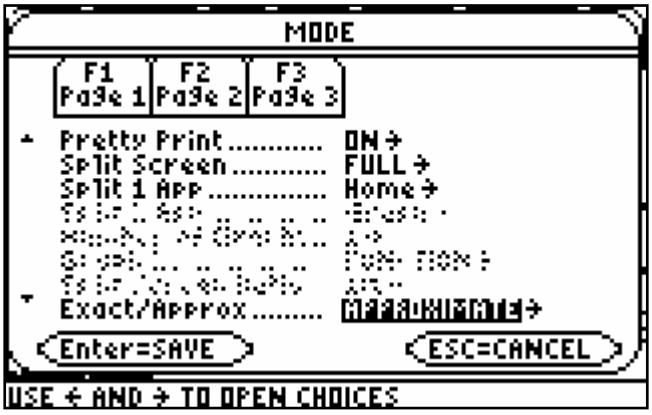
As an example, consider the formula for the volume of a cylinder $V = \pi r^2 h$.

Define a function $v(r, h)$ as $\pi r^2 h$.

Evaluate $v(r, h)$ for various values of *r* and *h*.



If you would like the answer displayed as an approximate value, rather than a multiple of π, you can select the **MODE** menu, scroll down to **Exact/Approx**, and select **APPROXIMATE**.



Tech Tip: when you are finished with the CAS, it is wise to return the display mode to **AUTO**.